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SEARCH, STICXY PRICES, AND INFLATION WITH CONSUMER DIFFERENCES

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## Search. Sticky Prices. and Inflation With Consumer Differences Peter A. Diamond ${ }^{1}$

This paper continues analysis of the sticker cost approach to equilibrium with sticky prices, considering an economy with two types of consumers rather than one. ${ }^{2}$ With the menu cost approach, a change in price by the firm changes the price of all following transactions. With the sticker cost approach, there is a separate price attached to each unit in inventory. This alternative reflects actual practice for some commodities where there is a large distribution of units of inventory available for inspection with prices attached. For example, it is common in second hand book stores in the U.S. Before electronic cashiers, it was common in supermarkets. For a time, Yugoslavia had a law forbidding changes of sticker prices for some goods. Moreover, this assumption avoids a difficult problem in equilibrium modelling with the menu cost assumption - the relative timing of price changes of different firms. By assuming a constant cost per commodity for price changes, all firms

[^0]will behave the same, continuously adjusting the price on newly priced goods and repricing the lowest price goods in inventory. This paper explores the comparative statics of steady economy wide inflation in a market with consumer search and optimal price setting by firms.

In the absence of inflation, in an industry with search by identical consumers seeking to purchase a single unit, prices are set equal to the reservation price of consumers, which in turn is equal to the utility of consumption (measured in money). Thus there is no consumer surplus. If firms can not change the nominal sticker price placed on a good, consumers will gain from the presence of previously priced goods that have their real prices lowered by inflation. This will lower reservation prices, lowering also the real price set on newly priced goods. While inflation will add to the expected delay in finding the good, this is of little consequence when consumer surplus is small. Thus consumers gain from steady inflation (Diamond, 1991). With sufficient inflation the delay factor becomes important, lowering consumer welfare with further increases in inflation.

When there are two types of consumers, for some parameter values there is a two-price equilibrium in the absence of inflation. ${ }^{3}$ Consumers with low utility from purchase receive no consumer surplus, buying goods that have been priced at their reservation price. But consumers with high utility do receive consumer surplus from the possibility of buying goods that have lower prices as a result of the presence of consumers with low utility. With inflation and

[^1]sticky nominal prices, those with low utility can only gain, since they receive no consumer surplus without inflation. For those with high utility, one needs to examine the behavior of the entire distribution of prices and the availability of goods. While inflation lowers the real prices on already priced goods, it is necessary to check that this effect is not offset by the shift in the fraction of goods that are priced with the low utility consumers in mind or by the delay in expected purchase. Since the impact of inflation on profitability is greater the lower the sale rate, the impact is larger on high priced goods. Thus one would expect a shift to low priced goods, further helping consumers. However, these effects can be offset by the decreased purchase rate that follows from the lower availability of goods coming from the need to offset the effect of inflation on profitability. ${ }^{4}$ We examine this question in the neighborhood of zero inflation by calculating the derivative of the steady state reservation price of high utility consumers with respect to inflation, evaluated at zero inflation. Given the complexity of the derived expression, the derivative was calculated for a wide range of parameter values. Table 1 reports parameter values.

The model has continuous time with a continuous flow of two types of new consumers into the market, each of whom seeks to purchase one unit provided the real price does not exceed the utility value of the good. There is utility discounting, but no explicit cost of search. On the firm side there is

[^2]free entry with identical firms and optimal price setting. The optimal price for a newly produced good is the maximum that one or the other type of consumer is willing to pay. Inflation produces the possibility of bargains from finding previously priced goods that have not yet been sold. The model assumes that the nominal interest rate rises one for one with the inflation rate. This assumption, appropriate for credit card purchases, is in contrast with a situation in which no interest is earned on the purchasing power being carried during the search process. It is assumed that the rate of meeting between consumers and commodities is a constant returns to scale function of the stocks of customers and inventory, with the probability of a contact being the same for each individual and each unit of inventory. In steady state equilibrium the flow of newly produced goods equals the exogenous flow of new customers. However, the stocks of goods in inventory and of searching customers adjust in response to the zero expected profit condition arising from free entry.

In sections 1 to 5 , the basic model is presented. Section 6 considers the special case when inflation is zero. Section 7 reports the derivative with respect to inflation of the reservation price of high utility consumers at zero inflation.

## 1. Matching Technology

It is assumed that there is a continuous flow of size $x$ of new customers into this market. Of these, the fraction $g^{\prime} /\left(l+g^{\prime}\right)$ have high utility of consumption, $u_{2}$. The rest have utility of consumption, $u_{1}$. Otherwise, customers are identical. This difference in preferences implies a difference
in reservation prices in the search process. Each customer seeks to purchase one unit of the commodity as long as the real price does not exceed his utility of consumption. Individual purchase decisions are made by comparing the price with the consumer's reservation price. We denote by $X$ the stock of customers actively searching in the market. Of these, the fraction $g /(1+g)$ have high reservation prices. Similarly, we denote by y the flow of newly produced commodities into inventory, and by $Y$ the stock of goods available in inventory. There is a matching technology which determines the flow rate of meetings as a function of the stocks of customers and inventory, $M(X, Y)$. We assume that $M$ has constant returns to scale with a strictly positive marginal contribution by each factor (when the other factor has a positive level), $M_{1}>0, M_{2}>0$. We are assuming that the distribution of prices on inventory has no effect on the rate of meetings, although it can affect the fraction of meetings that result in a purchase.

In steady state equilibrium the rate of sales equals the exogenous rate of arrival of new customers. Of the steady state inventory, the fraction $f$ is priced so that those with low willingness to pay are willing to purchase. All of inventory is priced so that those with high willingness to pay are willing to purchase. Equating new inventory flow with sales we have:

$$
\begin{equation*}
\mathrm{x}=\mathrm{M}(\mathrm{X}, \mathrm{Y})(\mathrm{f}+\mathrm{g}) /(\mathrm{l}+\mathrm{g}) \tag{1}
\end{equation*}
$$

That is, sales occur whenever a high reservation price customer finds a unit of inventory and in the fraction $f$ of meetings with inventory of a low reservation price customer.

We assume that each customer and each unit of inventory experience the same flow probabilities of a match and so experience the arrivals of transaction opportunities as Poisson processes. We denote these arrival rates of transactions for customers and inventory by $a_{i}$ and $b_{i}$ with $b_{1}$ the rate for low priced goods and $b_{2}$ the rate for high priced goods. In steady state equilibrium these must satisfy

$$
\begin{array}{ll}
a_{1}=\mathrm{fM} / X, & a_{2}=M / X  \tag{2}\\
b_{1}=M / Y, & b_{2}=M g /(Y(1+g))
\end{array}
$$

With constant returns to scale, we have

$$
\begin{equation*}
1=M\left(a_{2}^{-1}, b_{1}^{-1}\right) \tag{3}
\end{equation*}
$$

Given the arrival rates of transactions, we can relate the relative sizes of stocks of customers of the two types to their relative flows.

$$
\begin{equation*}
\mathrm{g}=\mathrm{fg} \mathrm{~g}^{\prime} \tag{4}
\end{equation*}
$$

## 2. The Distribution of Prices

As we will note below, firms will price newly produced goods at the reservation prices of customers; there is no reason for prices of newly produced
goods except at these two prices. We solve separately for the two distributions of prices on existing inventory coming from the two real initial prices. Each distribution of prices on goods currently in the market reflects the constant arrival rate of goods whose real prices decay exponentially at the inflation rate, $\pi(\pi>0)$, with the quantity of goods still remaining on the market at any given price also declining exponentially at the arrival rate of sales, $b_{i}$. Thus at any time the distributions of real prices in the market have positive densities between 0 and $P_{i}$, the prices set on newly produced goods. Consider any real price, s, in this interval. Purchases reduce the fraction of goods with prices below $s$ at the rate $b_{i} F_{j}(s)$ where $F_{j}$ are the distributions of prices in the market coming from the different initial prices. Inflation adds to the stock of goods with real prices below s at the rate $\pi s f_{j}(s)$ where $f_{j}$ is the density of prices. Equating these two flows, the steady state density of commodities with real prices satisfies

$$
\begin{equation*}
0<s<\mathrm{p}_{1} \quad \pi s \mathrm{f}_{2}(\mathrm{~s})=\mathrm{b}_{1} \mathrm{~F}_{2}(\mathrm{~s}) \quad \pi s \mathrm{f}_{1}(\mathrm{~s})=\mathrm{b}_{1} \mathrm{~F}_{1}(\mathrm{~s}) \tag{5}
\end{equation*}
$$

$$
P_{1}<s<P_{2} \quad \pi s f_{2}(s)=b_{2} F_{2}(s)
$$

Solving these differential equations we have:

$$
\begin{align*}
& F_{1}(s)=\left(s / P_{1}\right)^{b_{1} / \pi} \\
& F_{2}(s)= \begin{cases}\left(s / p_{1}\right)^{b_{1} / \pi}\left(p_{1} / p_{2}\right)^{b_{2} / \pi} & 0<s<p_{1} \\
\left(s / p_{2}\right)^{b_{2} / \pi} & P_{1}<s<p_{2}\end{cases} \tag{6}
\end{align*}
$$

These distributions are homogeneous of degree 0 in $b_{1}, b_{2}$ and $\pi$, since proportional changes in these variables are equivalent to a change in the units in which time is measured.

For consumer search we need to know the mean price on acceptable goods. For those selecting only from goods priced at $p_{1}$ or below, the mean price of goods on the market, $m_{1}$, (and so of transactions) equals the mean of the distribution $\mathrm{F}_{1}$ :

$$
\begin{equation*}
\mathrm{m}_{1}=\mathrm{b}_{1} \mathrm{p}_{1} /\left(\mathrm{b}_{1}+\pi\right) \tag{7}
\end{equation*}
$$

For those willing to buy any unit in inventories, the mean is the average of the means from distributions above and below $p_{1}$ with weights equal to the relative stocks of goods still on the market. The mean of goods priced above $P_{1}$ is equal to

$$
\begin{equation*}
\mathrm{m}_{2}=\left(\mathrm{b}_{2} /\left(\mathrm{b}_{2}+\pi\right)\right)\left(\left(\mathrm{p}_{2}^{\mathrm{b}_{2} / \pi+1}-\mathrm{p}_{1}^{\mathrm{b}_{2} / \pi+1}\right) /\left(\mathrm{p}_{2}{ }^{b_{2} / \pi}-\mathrm{p}_{1} \mathrm{~b}_{2} / \pi\right)\right) \tag{8}
\end{equation*}
$$

The mean transaction price satisfies

$$
\begin{equation*}
m_{2}=f m_{1}+(1-f) m_{2}^{\prime} \tag{9}
\end{equation*}
$$

where $f$ is the fraction of goods on the market at low price.

## 3. Consumer Search

We assume that the purchasing power held by customers while searching is earning the going rate of interest in the economy and that the real rate of interest in the economy is constant. Thus we assume that the nominal rate increases point for point with the inflation rate:

$$
\begin{equation*}
i=\pi+r \tag{10}
\end{equation*}
$$

where $i$ and $r$ are the nominal and real interest rates, respectively. This as sumption fits with payment by check or credit card rather than currency. We denote by $V_{i}$ the asset value of being a customer in the search market with utility $u_{i}$. We assume that the real rate of utility discount on the utility from consuming this good is equal to the real rate of interest in the economy earned on purchasing power. We also assume that utility is linear in income
available to spend on other goods. Thus we can use the standard dynamic programming framework for describing consumer search. We denote by $p_{i}{ }^{*}$ the reservation price of a consumer with utility $u_{i}$. The reservation price is equal to the utility from consuming the commodity less the value of continuing to search for later consumption:

$$
\begin{equation*}
p_{i}^{*}=u_{i}-v_{i} \tag{11}
\end{equation*}
$$

The dynamic programming approach implies that the real interest rate times the value of a position equals the arrival rate of transactions times the expected capital gain from a transaction.

$$
\begin{equation*}
r V_{i}=a_{i}\left(u_{i}-v_{i}-m_{i}\right) \tag{12}
\end{equation*}
$$

Dropping the distinction between the prices set on newly priced goods and the reservation price, we have two equations for the prices from consumer search behavior:

$$
\begin{equation*}
r\left(u_{i}-p_{i}\right)=a_{i}\left(p_{i}-m_{i}\right) \tag{13}
\end{equation*}
$$

The combination of a positive inflation rate and the only cost of search being delay in gratification implies that the reservation price is strictly less than the utility of consuming the good, $u_{i}$. The addition of an explicit search cost would raise the possibility that $u_{i}$ is the reservation price, rather than a value derived from the comparison of purchasing today with purchasing in the
future. Note from (11) that the reservation price of a consumer is a sufficient statistic for the expected utility of shopping, with $\mathrm{V}_{\mathrm{i}}$ decreasing in $p_{i}{ }_{i}$.

## 4. Pricing and Entry

We assume that the real cost of producing a unit for sale is c. This cost, like the utility of consumption, is rising in nominal terms at the rate, $\pi$. We assume that there is no setup cost to entering this market. With free entry and identical firms, the expected real discounted profit from producing a good for sale will equal $c$. When the firm produces a unit of the good it attaches a nominal price to the unit and is not allowed to revise that price in the future. No firm will set a price higher than the current reservation price, $P_{2}$. To do so would simply introduce a period when a good was sitting in inventory, not available for sale. This would lose the real interest rate on the real cost of production that has already taken place, even though there was no loss from inflation while waiting for the reservation price to rise to the level of price that has been set on the commodity. Given this fixed nominal price we can calculate the expected present discounted value of profit from the sale of this commodity using the usual dynamic programming approach. With price fixed in nominal terms, the equation is stated in nominal terms. $W(p)$ is used to denote the value of a newly produced commodity for sale that is priced at $p$. The form of $W$ depends on the price level since the rate of sale depends on price. Considering low prices first we have for

$$
\begin{equation*}
p \leq p_{1}: \quad i W=b_{1}(p-W) \tag{14}
\end{equation*}
$$

Since $W$ is strictly decreasing in $p$, no newly produced good will be priced below $P_{1}$ the lowest reservation price. For prices between $P_{1}$ and $P_{2}$ the arrival rate of a sale is $b_{2}$. In addition there is a gain from the decrease in the time until the sale rate rises to $b_{1}$ if the good is not yet sold. One way to calculate value in this region is to calculate value if the sale rate didn't change and then add the expected present discounted value of the gain from a higher sale rate once the price is below $P_{1}$. The probability of no sale for long ènough for the real price to fall from $p$ to $p_{1}$ is $\left(p_{1} / p\right)^{b_{2} / \pi}$. The discount factor for that length of time is $\left(p_{1} / p\right)^{r / \pi}$. Thus we have
$W(p)= \begin{cases}b_{1} p\left(r+\pi+b_{1}\right) & p<p_{1} \\ b_{2} p /\left(r+\pi+b_{2}\right)+ \\ \left(\left(p_{1} / p\right)\left(b_{2}+r\right) / \pi_{p_{1}}\left(b_{1}-b_{2}\right)(r+\pi)\right) /\left(\left(r+\pi+b_{1}\right)\left(r+\pi+b_{2}\right)\right)\end{cases}$

With free entry $W\left(p_{1}\right)$ and $W\left(p_{2}\right)$ must both equal the cost of production, $c$ with W no higher at any other value. Converting the nominal interest rate into a real rate plus the inflation rate, the zero profit conditions can be written as:

$$
\begin{align*}
c= & b_{1} p_{1} /\left(r+\pi+b_{1}\right)  \tag{16}\\
c= & b_{2} p_{2} /\left(r+\pi+b_{2}\right)+ \\
& \left(\left(p_{1} / p_{2}\right)\left(b_{2}+r / \pi\right) p_{1}\left(b_{1}-b_{2}\right)(r+\pi)\right) /\left(\left(r+\pi+b_{1}\right)\left(r+\pi+b_{2}\right)\right)
\end{align*}
$$

The markups over cost depend upon the real interest rate, the inflation rate, and the arrival rates of customers.

In order to have an equilibrium, $W$ as a function of $p$ must appear as in Figure 1. This requires that $W$ be decreasing in $p$ for $p$ just above $p_{1}$. Differentiating $W$ in (15) with respect to $p$ for values of $p$ above $p_{1}$, we have:

$$
\begin{equation*}
W^{\prime}(p)=\left(b_{2} / \pi\right)-\left(\left(b_{2}+r\right) / \pi\right)(W(p) / p) \tag{17}
\end{equation*}
$$

Evaluating the derivative at $p_{1}$, the negativity of the derivative is equivalent to

$$
\begin{equation*}
r /(r+\pi)>b_{2} / b_{1}=f g^{\prime} /\left(1+f g^{\prime}\right) \tag{18}
\end{equation*}
$$

This condition is satisfied for $\pi$ sufficiently small, provided we have a proper equilibrium $(0<f<1)$ at $\pi=0$. To confirm the rest of the shape shown in Figure 1 , we differentiate (17) again

Figure 1


$$
\begin{align*}
\mathrm{W}^{\prime \prime}(\mathrm{p})=- & \left(\left(\mathrm{b}_{2}+\mathrm{r}\right) / \pi\right)\left(\left(\mathrm{pW} \mathrm{~W}^{\prime}(\mathrm{p})-\mathrm{W}(\mathrm{p})\right) / \mathrm{p}^{2}\right.  \tag{19}\\
= & -\left(\left(\mathrm{b}_{2}+r\right) / \pi\right)\left(\left(\mathrm{b}_{2} \mathrm{p} / \pi\right)-\left(\left(\mathrm{b}_{2}+r+\pi\right) / \pi\right) W(\mathrm{p})\right) \\
= & \left(\left(\mathrm{b}_{2}+r\right) / \pi\right)\left(\left(\mathrm{p}_{1} / \mathrm{p}\right)\left(\mathrm{b}_{2}+r\right) / \pi \mathrm{p}_{1}\left(\mathrm{~b}_{1}-\mathrm{b}_{2}\right)(\mathrm{r}+\pi)\right) / \\
& \left(\left(\mathrm{r}+\pi+\mathrm{b}_{1}\right)\left(\mathrm{r}+\pi+\mathrm{b}_{2}\right)\right)>0
\end{align*}
$$

Thus $W$ is convex between $P_{1}$ and $\mathrm{P}_{2}$.

## 5. Equilibrium

In order to examine prices, we consider the model in terms of ten endogenous variables: $a_{1} a_{2} b_{1}, b_{2}, f, g, m_{1}, m_{2}, P_{1}$ and $P_{2}$. Without restriction on the search technology there is no assurance that there will be an equilibrium with positive production, even with the cost of production less than the utility of consumption $\left(c<u_{i}\right)$. What is needed is a search technology that permits $b_{i}$ to be sufficiently large that firms can cover costs even with a high inflation rate. For example, Cobb-Douglas search technology ensures existence. Moreover, we are interested in equilibria where both prices are being set on newly produced goods. This requires $0<\mathrm{f}<1$. Rather than analyze this system in detail, we focus on the question of whether a small amount of inflation lowers the higher reservation price, $\mathrm{P}_{2}$. To answer this question, we first solve for equilibrium with zero inflation. We then calculate the derivatives of $f$ and $p_{2}$ with respect to $\pi$, evaluated at the equilibrium with zero inflation.

## 6. Stable Prices

Since we are particularly interested in the behavior of the economy for low inflation rates, we begin analysis of this economy by solving the system of equations for zero inflation. Setting $\pi=0$, we have

$$
\begin{align*}
& p_{1}=u_{1} \\
& p_{2}=\left(r u_{2}+a_{2} f u_{1}\right) /\left(r+a_{2} f\right) \\
& 1=M\left(a_{2}^{-1}, b_{1}^{-1}\right)  \tag{20}\\
& b_{1}=r c /\left(u_{1}-c\right) \\
& f=r\left(u_{1}-c\right) /\left(r g^{\prime}\left(u_{2}-u_{1}\right)-\left(u_{1}-c\right) a_{2}\right)
\end{align*}
$$

For this to be a two price distribution, we need $0<f<1$. With $u_{2}>u_{1}>c$, both of these conditions are satisfied if and only if

$$
\begin{equation*}
u_{1}<\left(r g^{\prime} u_{2}+\left(a_{2}+r\right) c\right) /\left(r g^{\prime}+a_{2}+r\right) \tag{21}
\end{equation*}
$$

Naturally, we assume that this condition holds.

For calculations, we assume that the matching technology is Cobb Douglas:

$$
\begin{equation*}
M=A X^{\alpha} Y^{1-\alpha} \tag{22}
\end{equation*}
$$

This technology implies

$$
\begin{align*}
& a_{2}^{\alpha}=A\left(\left(u_{1}-c\right) / r c\right)^{1-\alpha} \\
& f=\left(u_{1}-c\right) /\left(g^{\prime}\left(u_{2}-u_{1}\right)-(A / r)^{1 / \alpha}\left(u_{1}-c\right)^{1 / \alpha}\right) \tag{23}
\end{align*}
$$

We restrict analysis to parameters for which $0<£<1$.

## 7. Response to Inflation

In the Appendix are the details of the differentiation of $p_{2}$ with respect to $\pi$, evaluated at $\pi=0$. The set of equations can be reduced to three equations, denoted $A, B$, and $C$ in the appendix. The details of differentiations are shown there. Since the expression is too long to be analytically useful, calculations of the derivatives for different sets of parameters are shown in Table 1.

Calculations of the derivative relative to different parameters shows that the derivative can be of either sign. Further calculations will illuminate the determinants of the pattern.

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Appendix

$$
\begin{aligned}
& A \equiv 1-M\left\{1 / a_{2}, f g^{\prime} /\left[\left(1+f g^{\prime}\right) b_{2}\right]\right) \\
& B \equiv\left(r+f a_{2}\right) z p_{2}-r u_{1}-f a_{2} m_{1} \\
& c \equiv\left(r+\pi+b_{2}\right) c / p_{2}-b_{2}-\left[(r+\pi) b_{2} z^{\left(b_{2}+r+\pi\right) / \pi}\right] /\left[(r+\pi) f g^{\prime}+\left(1+f g^{\prime}\right) b_{2}\right] \\
& z=\left[c\left((r+\pi) f g^{\prime}+\left(1+f g^{\prime}\right) b_{2}\right)\right] /\left[p_{2}\left(1+f^{\prime}\right) b_{2}\right] \\
& m_{1}=\left[\left(1+f g^{\prime}\right) b_{2} P_{2} z\left\{f, b_{2}, P_{2}, \pi\right)\right] /\left[\left(1+f g^{\prime}\right) b_{2}+\pi f g^{\prime}\right] \\
& m_{2}^{\prime}=\left[b_{2} p_{2}\left(1-z()^{\left(b_{2}+\pi\right) / \pi}\right)\right] /\left[\left(b_{2}+\pi\right)\left(1-z()^{b_{2} / \pi}\right)\right] \\
& m_{2}=f m_{1}( \}+(1-f) m_{2}^{\prime}\{ \} \\
& a_{2}=r\left(u_{2}-p_{2}\right) /\left(p_{2}-m_{2}(1)\right. \\
& \alpha=M_{X} / a_{2}, \quad 1-\alpha=M_{Y} f^{\prime} /\left[b_{2}\left(1+f_{g}^{\prime}\right)\right] \\
& A_{f}=M_{X}\left\{\delta a_{2} / \delta f\right\} / a_{2}^{2}-g^{\prime} M_{Y} /\left[b_{2}\left(1+f g^{\prime}\right)^{2}\right]=\alpha\left\{\delta a_{2} / \delta f\right\} / a_{2}-g^{\prime}(1-\alpha) /\left[f g^{\prime}\left(1+f g^{\prime}\right)\right] \\
& A_{b}=M_{X}\left(\delta a_{2} / \delta b\right\} / a_{2}{ }^{2}+M_{Y} f^{\prime} /\left[b_{2}{ }^{2}\left(1+f g^{\prime}\right)\right]=\alpha\left\{\delta a_{2} / \delta b\right\} / a_{2}+(1-\alpha) / b_{2} \\
& A_{p}=M_{X}\left\{\delta a_{2} / \delta p_{2}\right\} / a_{2}^{2}=\alpha\left\{\delta a_{2} / \delta p_{2}\right\} / a_{2} \\
& A_{\pi}=M_{X}\left\{\delta a_{2} / \delta \pi\right\} / a_{2}^{2}=\alpha\left\{\delta a_{2} / \delta \pi\right\} / a_{2} \\
& B_{f}=\left(r+f a_{2}\right) p_{2}(\delta z / \delta f)-f a_{2}\left(\delta \mathrm{~m}_{1} / \delta f\right) \\
& B_{b}=\left(r+f a_{2}\right) p_{2}(\delta z / \delta b)-f a_{2}\left(\delta m_{1} / \delta b\right) \\
& B_{p}=\left(r+f a_{2}\right) z+\left(r+f a_{2}\right) p_{2}\{\delta z / \delta p\}-f a_{2}\left\{\delta m_{1} / \delta p\right\} \\
& \mathrm{B}_{\pi}=\mathrm{fa}_{2} \mathrm{p}_{2}\{\delta \mathrm{z} / \delta \pi\}-\mathrm{fa}_{2}\left\{\delta \mathrm{~m}_{1} / \delta \pi\right\} \\
& C_{f}=0 \\
& C_{b}=c / p_{2}-1 \\
& C_{p}=-\left(r+b_{2}\right) c / p_{2}{ }^{2} \\
& C_{\pi}=c / p_{2}
\end{aligned}
$$

$$
\begin{aligned}
& \delta z / \delta f=\mathrm{crg}^{\prime} /\left[\mathrm{p}_{2}\left(1+\mathrm{fg}^{\prime}\right)^{2} \mathrm{~b}_{2}\right] \\
& \delta z / \delta b=-\operatorname{crfg}^{\prime} /\left[p_{2}\left(1+\mathrm{fg}^{\prime}\right) \mathrm{b}_{2}{ }^{2}\right] \\
& \delta z / \delta p=-z / p_{2} \\
& \delta z / \delta \pi=\mathrm{cfg}^{\prime} /\left[\mathrm{p}_{2}\left(1+\mathrm{fg}^{\prime}\right) \mathrm{b}_{2}\right] \\
& \delta \mathrm{m}_{1} / \delta \mathrm{f}=\mathrm{crg}{ }^{\prime} /\left[\left(1+\mathrm{fg}^{\prime}\right)^{2} \mathrm{~b}_{2}\right] \\
& \delta \mathrm{m}_{1} / \delta \mathrm{b}=-\mathrm{crfg}{ }^{\prime} /\left[\left(1+\mathrm{fg}^{\prime}\right) \mathrm{b}_{2}{ }^{2}\right] \\
& \delta \mathrm{m}_{1} / \delta \mathrm{p}=0 \\
& \delta \mathrm{~m}_{1} / \delta \pi=-\left(\mathrm{p}_{1}-\mathrm{c}\right) \mathrm{fg} \mathrm{~g}^{\prime} /\left[\left(1+\mathrm{fg}^{\prime}\right) \mathrm{b}_{2}\right] \\
& \delta \mathrm{m}_{2}{ }^{\prime} / \delta \mathrm{f}=0 \\
& \delta \mathrm{~m}_{2}{ }^{\prime} / \delta \mathrm{b}_{2}=0 \\
& \delta \mathrm{~m}_{2}{ }^{\prime} / \delta \mathrm{p}_{2}=1 \\
& \delta \mathrm{~m}_{2}{ }^{\prime} / \delta \pi=-\mathrm{P}_{2} / \mathrm{b}_{2} \\
& \delta \mathrm{~m}_{2} / \delta \mathrm{f}=\mathrm{u}_{1}-\mathrm{P}_{2}+\operatorname{crfg}^{\prime} /\left[\left(1+\mathrm{fg}^{\prime}\right)^{2} \mathrm{~b}_{2}\right] \\
& \delta \mathrm{m}_{2} / \delta \mathrm{b}=-\mathrm{crf}^{2} \mathrm{~g}^{\prime} /\left[\left(1+\mathrm{fg}^{\prime}\right) \mathrm{b}_{2}{ }^{2}\right] \\
& \delta \mathrm{m}_{2} / \delta \mathrm{p}=1-\mathrm{f} \\
& \delta \mathrm{~m}_{2} / \delta \pi=-\mathrm{f}\left(\mathrm{p}_{1}-\mathrm{c}\right) \mathrm{fg}^{\prime} /\left[\left(1+\mathrm{fg}^{\prime}\right) \mathrm{b}_{2}\right]-(1-\mathrm{f}) \mathrm{p}_{2} / \mathrm{b}_{2} \\
& \delta a_{2} / \delta f=a_{2}\left(u_{1}-p_{2}+c r f g^{\prime} /\left[\left(1+\mathrm{fg}^{\prime}\right) \mathrm{b}_{2}\right]\right) /\left[f\left(\mathrm{p}_{2}-\mathrm{u}_{1}\right)\right] \\
& \delta a_{2} / \delta b=-a_{2} c r f g^{\prime} /\left[\left(p_{2}-u_{1}\right)\left(1+\mathrm{fg}^{\prime}\right) b_{2}^{2}\right] \\
& \delta a_{2} / \delta p_{2}=\left[-r-a_{2} f\right] /\left[f\left(p_{2}-u_{1}\right)\right] \\
& \delta a_{2} / \delta \pi=-a_{2}\left(u_{1}-c\right) f^{2} g^{\prime} /\left[\left(1+f g^{\prime}\right) b_{2} f\left(p_{2}-u_{1}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& D \equiv\left|\begin{array}{lll}
A_{f} & A_{b} & A_{p} \\
B_{f} & B_{b} & B_{p} \\
C_{f} & C_{b} & C_{p}
\end{array}\right| \\
& N_{p} \equiv\left|\begin{array}{lll}
A_{f} & A_{b} & A_{\pi} \\
B_{f} & B_{b} & B_{\pi} \\
C_{f} & C_{b} & C_{\pi}
\end{array}\right| \\
& D=A_{f}\left(B_{b} C_{p}-B_{p} C_{b}\right)-A_{b} C_{p} B_{f}+A_{p} B_{f} C_{b} \\
& N_{p}=A_{f}\left(B_{b} C_{\pi}-B_{\pi} C_{b}\right)-A_{b} C_{\pi} B_{f}+A_{\pi} B_{f} C_{b} \\
& d p_{2} / d \pi=-N_{p} / D \\
& N_{f} \equiv\left|\begin{array}{lll}
A_{\pi} & A_{b} & A_{p} \\
B_{\pi} & B_{b} & B_{p} \\
C_{\pi} & C_{b} & C_{p}
\end{array}\right| \\
& d f / d \pi=-N_{f} / D
\end{aligned}
$$

Tabile 1

| r | C | ul | u 2 | $g \mathrm{P}$ | A | alpha | f | dP | df |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.10 | 1.00 | 3.00 | 3.57 | 5.25 | 0.001 | 1.00 | 0.673 | 0.0243 | -29.8 |
| 0.10 | 1.00 | 2.96 | 3.57 | 5.25 | 0.001 | 1.00 | 0.616 | -0.0107 | -25.6 |
| 0.10 | 1.00 | 3.00 | 3.60 | 5.25 | 0.001 | 1.00 | 0.639 | -0.0003 | -27.2 |
| 0.10 | 1.00 | 3.00 | 3.57 | 5.50 | 0.001 | 1.00 | 0.642 | 0.0234 | -28.5 |
| 0.10 | 1.00 | 3.00 | 3.57 | 5.25 | 0.001 | 0.50 | 0.668 | 0.0006 | -30.1 |
| 0.10 | 1.00 | 2.96 | 3.57 | 5.25 | 0.001 | 0.50 | 0.612 | -0.0001 | -25.7 |
| 0.10 | 1.00 | 3.00 | 3.61 | 5.25 | 0.001 | 0.50 | 0.625 | -0.0001 | -26.7 |
| 0.10 | 1.00 | 4.00 | 4.68 | 7.59 | 0.001 | 1.00 | 0.585 | 0.0195 | -31.0 |
| 0.10 | 1.00 | 3.98 | 4.68 | 7.59 | 0.001 | 1.00 | 0.564 | -0.0038 | -29.1 |
| 0.10 | 1.00 | 4.00 | 4.70 | 7.59 | 0.001 | 1.00 | 0.568 | -0.0024 | -29.4 |
| 0.10 | 1.00 | 4.00 | 4.68 | 7.84 | 0.001 | 1.00 | 0.566 | 0.0193 | -30.0 |
| 0.10 | 1.00 | 4.00 | 4.68 | 7.59 | 0.001 | 0.50 | 0.581 | 0.0010 | -31.4 |
| 0.10 | 1.00 | 3.96 | 4.68 | 7.59 | 0.001 | 0.50 | 0.542 | -0.0004 | -27.6 |
| 0.10 | 1.00 | 4.00 | 4.71 | 7.59 | 0.001 | 0.50 | 0.557 | -0.0001 | -29.0 |
| 0.10 | 1.00 | 4.00 | 4.68 | 7.84 | 0.001 | 0.50 | 0.563 | 0.0009 | -30.4 |

Table 2. Equilibrium with inflation rates of 0 and .1 $g^{\prime}=1, A=1$, alpha=. $5, r=.1$

| $p_{1}(0)$ | $p_{1}(.1)$ | $p_{2}(0)$ | $p_{2}(.1)$ | $f(0)$ | $f(.1)$ | $u 1$ | $u 2$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1.0124 | 1.0122 | 1.1804 | 1.1568 | .07385 | .06478 | 1.0124 | 1.1958 |
| 1.0170 | 1.0164 | 1.1789 | 1.1550 | .10505 | .08935 | 1.0170 | 1.2079 |
| 1.0381 | 1.0328 | 1.1083 | 1.1475 | .54306 | .19595 | 1.0381 | 1.2536 |
| 1.0221 | 1.0214 | 1.4129 | 1.3180 | .05651 | .05660 | 1.0221 | 1.4617 |
| 1.0526 | 1.0457 | 1.3368 | 1.3071 | .18499 | .12898 | 1.0526 | 1.6133 |
| 1.0675 | 1.0551 | 1.2147 | 1.3026 | .45912 | .15975 | 1.0675 | 1.6709 |
| 1.0117 | 1.0113 | 1.0800 | 1.0758 | .17039 | .12676 | 1.0117 | 1.0936 |
| 1.0207 | 1.0187 | 1.0591 | 1.0723 | .53949 | .22994 | 1.0207 | 1.1021 |
| 1.0065 | 1.0064 | 1.0382 | 1.0375 | .20491 | .14565 | 1.0065 | 1.0425 |

Figure 1


## HE




[^0]:    ${ }^{1}$ I am grateful to Jackie Rosner and Dimitri Vayanos for research assistance and to NSF for research support.
    ${ }^{2}$ Previous sticker cost analyses are Diamond (1991) and Diamond and Felli (1990). These contrast with menu cost studies, such as Barro (1972), Benabou (1988,1989), Caballero and Engel (1989), Caplin and Spulber (1987), Caplin and Leahy (1989), Sheshinski and Weiss (1977).

[^1]:    ${ }^{3}$ Without inflation, the model employed is essentially the same as in Diamond (1987).

[^2]:    ${ }^{4}$ This suggests the conjecture that all consumers are better off from low inflation if consumers can sample goods at a fixed (stochastic) rate. Calculations just completed contradict this conjecture.

